Spiral cylindrique avec courbes terminales en arc de cercle

Perturbations causées par l'inertie du spiral

Caractéristiques du spiral

Référence :C:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm \quad TOL := 10^{-12}$

Elinvar $\quad \rho_s = 8\times 10^3\ m^{-3}\cdot kg \quad E = 1.7\times 10^{11}\,Pa \quad G = 6.538\times 10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s\cdot 360\cdot deg \quad \psi_0 = 3.654\times 10^3\,deg \quad L := R_0\cdot\psi_0 \quad L = 318.872\,mm$

Forme du spiral en fonction de l'élongation du balancier $\quad \psi(\theta) := \psi_0 + \theta \quad R(\theta) := \dfrac{L}{\psi(\theta)}$

$$x(\alpha,\theta) := R(\theta)\cdot cos(\alpha) \quad y(\alpha,\theta) := R(\theta)\cdot sin(\alpha) \quad s(\alpha,\theta) := R(\theta)\cdot\alpha$$

Calcul de la variation du moment d'inertie pour la partie cylindrique

$$\rho(\alpha,\theta) := \sqrt{R(\theta)^2 + \frac{R(\theta)^4}{s(\alpha,\theta)^2}} \qquad \rho(\alpha,\theta) := R(\theta)\cdot\sqrt{1+\frac{1}{\alpha^2}}$$

$$J_s(\theta) := \frac{m_s}{L^3}\cdot\int_0^{\psi(\theta)} \rho(\alpha,\theta)^2\cdot s(\alpha,\theta)^2\cdot R(\theta)\,d\alpha \qquad J_s(0) = 6.466\,mg\cdot cm^2 \qquad J_s(\theta_0) = 5.606\,mg\cdot cm^2$$

$$J_s(\theta) := \frac{m_s\cdot L^2}{\psi(\theta)^2}\cdot\left(\frac{1}{3}+\frac{1}{\psi(\theta)^2}\right) \qquad J_s(0) = 6.466\,mg\cdot cm^2 \qquad J_s(\theta_0) = 5.606\,mg\cdot cm^2$$

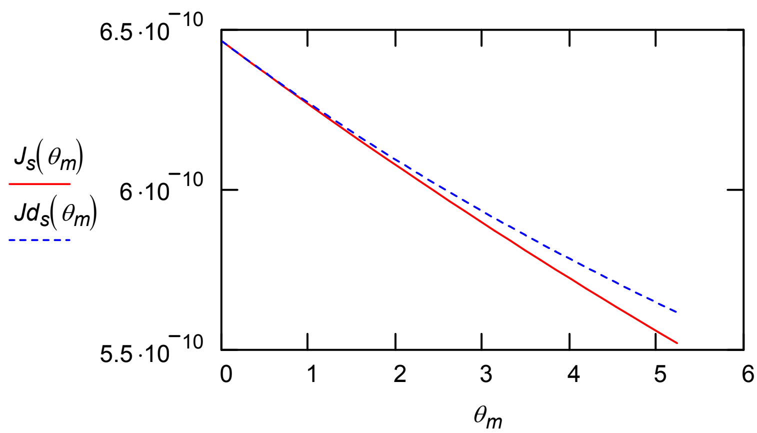
Par développement en série $\quad n := 0, 2 .. 6$

$$A(\theta,n) := \frac{n+1}{3\cdot\psi(\theta)^{n+2}}\cdot\left[1+\frac{(0.5\cdot n+1)\cdot(n+3)}{\psi(\theta)^2}\right]$$

$$\mathbf{A}(\theta) := (A(\theta,0)\quad 0\quad A(\theta,2)\quad 0\quad A(\theta,4)\quad 0\quad A(\theta,6))^T \qquad Jd_s(\theta) := m_s\cdot L^2\cdot\sum_n\left(\mathbf{A}(\theta)_n\cdot\theta^n\right)$$

$$\mathbf{A}(\theta_0)^T = \left(7.111\times 10^{-5}\quad 0\quad 4.555\times 10^{-8}\quad 0\quad 0\quad 0\quad 0\right)$$

$$\theta_m := 0\cdot deg, 10\cdot deg .. 300\cdot deg$$



Perturbation de marche

Moment d'inertie du balancier $\qquad J_b = 434.68\,mg\cdot cm^2$

Calcul de la perturbation de marche par intégration numérique

$$\delta_{Js}(\theta_0) := \frac{1}{2\cdot\pi\cdot J_b}\cdot\int_0^{\pi} J_s(\theta_0\cdot cos(\varphi))\,d\varphi \qquad \delta_{Js}(\theta_0) = 7.4991\times 10^{-3} \qquad \mu_{Js}(\theta_0) := -86400\cdot\delta_{Js}(\theta_0)$$

$$\delta_{Js}(0) = 7.438\times 10^{-3} \qquad \mu_{Js}(\theta_0) = -648$$

Selon Haag:

$$f(\theta_0) := \sum_n \left[\frac{(n+1)!}{\left(\left(\frac{n}{2}\right)!\right)^2}\cdot\left(\frac{\theta_0}{2\cdot\psi_0}\right)^n\right] \qquad g(\theta_0) := \sum_n \left[\frac{(n+1)!\cdot\left(\frac{n}{2}+1\right)\cdot(n+3)}{\left(\left(\frac{n}{2}\right)!\right)^2}\cdot\left(\frac{\theta_0}{2\cdot\psi_0}\right)^n\right]$$

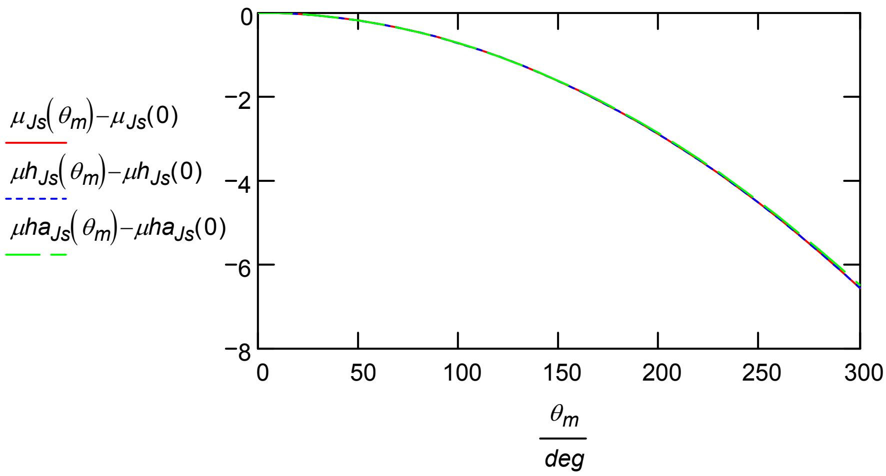
$$\delta h_{Js}(\theta_0) := \frac{m_s\cdot L^2}{6\cdot J_b\cdot\psi_0^2}\cdot\left(f(\theta_0)+\frac{g(\theta_0)}{\psi_0^2}\right) \qquad \delta h_{Js}(\theta_0) = 7.4991\times 10^{-3} \qquad \mu h_{Js}(\theta_0) := -86400\cdot\delta h_{Js}(\theta_0)$$

$$\delta h_{Js}(0) = 7.4376\times 10^{-3} \qquad \mu h_{Js}(\theta_0) = -648$$

Approximativement

$$\delta ha_{Js}(\theta_0) := \frac{m_s\cdot L^2}{6\cdot J_b\cdot\psi_0^2}\cdot\left(1+\frac{3}{2}\cdot\frac{\theta_0^2}{\psi_0^2}\right) \qquad \delta ha_{Js}(\theta_0) = 7.493\times 10^{-3} \qquad \mu ha_{Js}(\theta_0) := -86400\cdot\delta ha_{Js}(\theta_0)$$

$$\delta ha_{Js}(0) = 7.4321\times 10^{-3} \qquad \mu ha_{Js}(\theta_0) = -647$$



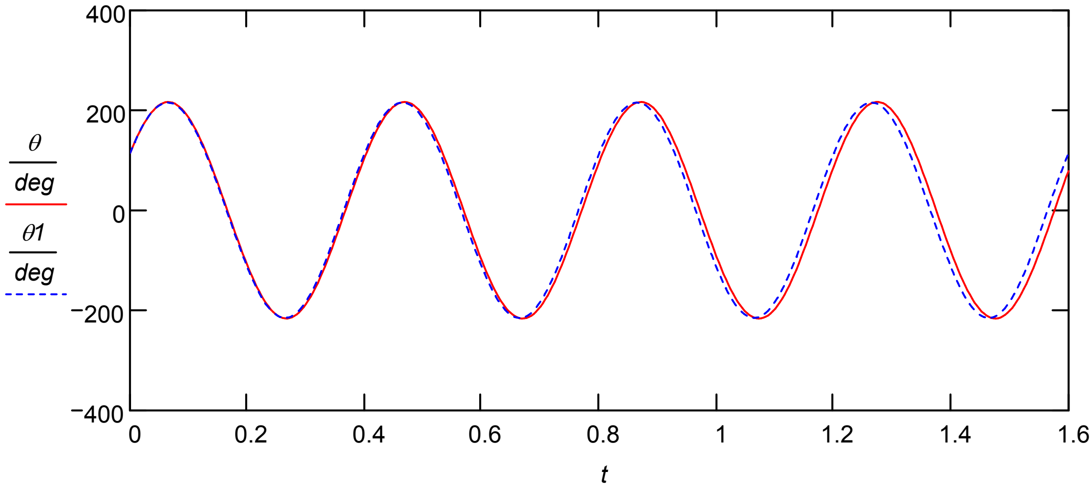
$$\mu_{Js}(0) = -642.61$$

Résolution de l'équation différentielle du mouvement

$$I(\theta) := J_b + J_s(\theta) \qquad b(\theta) := \frac{1}{2\cdot I(\theta)}\cdot\frac{d}{d\theta}J_s(\theta) \qquad c(\theta) := \frac{C}{I(\theta)} \qquad t_f := 4\cdot T_0\cdot\frac{1}{sec} \qquad t_f = 1.6$$

$$y := \begin{pmatrix} 2 \\ 50 \end{pmatrix} \qquad D(t,y) := \begin{bmatrix} y_1 \\ -b(y_0)\cdot(y_1)^2 - c(y_0)\cdot sec^2\cdot y_0 \end{bmatrix} \qquad \omega_0 := \omega_0\cdot sec \qquad D1(t,y) := \begin{pmatrix} y_1 \\ -\omega_0^2\cdot y_0 \end{pmatrix}$$

$$\Theta := rkfixe(y,0,t_f,400,D) \qquad t := \Theta^{\langle 0\rangle} \qquad \theta := \Theta^{\langle 1\rangle} \qquad Z := rkfixe(y,0,t_f,400,D1) \qquad \theta 1 := Z^{\langle 1\rangle}$$



Influence de l'inertie de courbes terminales circulaires

Courbe terminale de Phillips

$r_0 := 0.827 \cdot R_0 \qquad \beta_0 := 242.42 \cdot deg \qquad l_t := r_0 \cdot \beta_0 \qquad l_t = 17.5\,mm \qquad \varepsilon := \dfrac{l_t}{L} \qquad \varepsilon = 0.055 \qquad L_t := L + 2 \cdot l_t$

Inertie de la partie cylindrique du spiral

$$\psi(\theta) := \psi_0 + \theta \cdot \frac{L}{L_t} \qquad R(\theta) := \frac{L}{\psi(\theta)} \qquad \rho(\alpha, \theta) := R(\theta) \cdot \sqrt{1 + \frac{1}{\alpha^2}}$$

$$J_{sc}(\theta) := \frac{m_s}{L_t^{\,3}} \cdot \int_{l_t}^{L+l_t} \left(R(\theta)^2 \cdot s^2 + R(\theta)^4\right) ds \qquad J_{sc}\left(\theta_0\right) = 4.88\,mg \cdot cm^2$$

$$J_{sc}(\theta) := \frac{m_s \cdot L^5}{L_t^{\,3}} \cdot \left(\frac{1 + 3 \cdot \varepsilon + 3 \cdot \varepsilon^2}{3 \cdot \psi(\theta)^2} + \frac{1}{\psi(\theta)^4}\right) \qquad J_{sc}\left(\theta_0\right) = 4.88\,mg \cdot cm^2$$

Inertie de la courbe terminale externe $\qquad \varepsilon_t := \dfrac{l_t}{L_t} \qquad \beta_t(\theta) := \beta_0 + \varepsilon_t \cdot \theta \qquad r_t(\theta) := \dfrac{l_t}{\beta_t(\theta)}$

$$\rho(\alpha, \theta) := r_t(\theta) \cdot \sqrt{1 + \frac{4}{\alpha^2} \cdot \sin\left(\frac{\alpha}{2}\right)^2 - \frac{4}{\alpha} \cdot \sin\left(\frac{\alpha}{2}\right) \cdot \cos\left(\frac{\alpha}{2}\right)} \qquad s_t(\alpha, \theta) := r_t(\theta) \cdot \alpha$$

$$J_{st}(\theta) := \frac{m_s}{L_t^{\,3}} \cdot \int_0^{\beta_t(\theta)} \rho(\alpha, \theta)^2 \cdot s_t(\alpha, \theta)^2 \cdot r_t(\theta)\, d\alpha \qquad J_{st}\left(\theta_0\right) = 6.514 \times 10^{-4}\,mg \cdot cm^2$$

$$J_{st}(\theta) := m_s \cdot L_t^{\,2} \cdot \varepsilon_t^{\,5} \cdot \left[\frac{1}{3 \cdot \beta_t(\theta)^2} + 2 \cdot \left(\frac{1 + \cos\left(\beta_t(\theta)\right)}{\beta_t(\theta)^4}\right) - 4 \cdot \frac{\sin\left(\beta_t(\theta)\right)}{\beta_t(\theta)^5}\right] \qquad J_{st}\left(\theta_0\right) = 6.514 \times 10^{-4}\,mg \cdot cm^2$$

Inertie de la courbe terminale interne $\qquad \varepsilon_t := \dfrac{l_t}{L_t} \qquad \beta_t(\theta) := \beta_0 + \varepsilon_t \cdot \theta \qquad r_{t'}(\theta) := \dfrac{l_t}{\beta_t(\theta)}$

$$\rho2\left(\alpha_{t'}, \theta\right) := \left(R(\theta) - r_{t'}(\theta)\right)^2 + r_{t'}(\theta)^2 + 2 \cdot r_{t'}(\theta) \cdot \left(R(\theta) - r_{t'}(\theta)\right) \cdot \cos\left(\alpha_{t'}\right) \qquad s_{t'}\left(\alpha_{t'}, \theta\right) := L + l_t + r_{t'}(\theta) \cdot \alpha_{t'}$$

$$B(\theta) := \left(\frac{2 \cdot \varepsilon}{\beta_t(\theta) \cdot \psi(\theta)} - \frac{2 \cdot \varepsilon^2}{\beta_t(\theta)^2}\right) \qquad A(\theta) := \frac{1}{\psi(\theta)^2} - B(\theta)$$

$$J_{st'}(\theta) := \frac{m_s}{L_t^{\,3}} \cdot \int_0^{\beta_t(\theta)} \rho 2\left(\alpha_{t'}, \theta\right) \cdot s\left(\alpha_{t'}, \theta\right)^2 \cdot r_t(\theta)\, d\alpha_{t'} \qquad J_{st'}\left(\theta_0\right) = 5.28 \times 10^{-4}\, mg \cdot cm^2$$

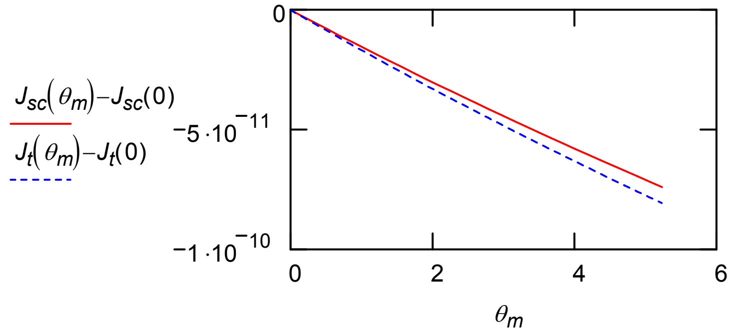
$$J_{st'}(\theta) := \frac{m_s \cdot L^2}{L_t^{\,3}} \cdot \int_{L_t - l_t}^{L_t} \left(A(\theta) + B(\theta) \cdot \cos\left(\frac{s_{t'} - L_t + l_t}{r_{t'}(\theta)}\right)\right) \cdot s_{t'}^{\,2}\, ds_{t'} \qquad J_{st'}\left(\theta_0\right) = 0.53\, mg \cdot cm^2$$

$$J_{st'}(\theta) := \frac{m_s \cdot r_{t'}(\theta) \cdot L^2}{L_t^{\,3}} \cdot \int_0^{\beta_t(\theta)} \left(A(\theta) + B(\theta) \cdot \cos\left(\alpha_{t'}\right)\right) \cdot \left(L_t - l_t + r_{t'}(\theta) \cdot \alpha_{t'}\right)^2\, d\alpha_{t'} \qquad J_{st'}\left(\theta_0\right) = 0.53\, mg \cdot cm^2$$

$$J_{st'}(\theta) := m_s \cdot L^2 \cdot \left[\frac{\varepsilon_t}{\beta_t(\theta)} \cdot B(\theta) \cdot \left[\left(1 - 2 \cdot \frac{\varepsilon_t^{\,2}}{\beta_t(\theta)^2}\right) \cdot \sin\left(\beta_t(\theta)\right) + 2 \cdot \frac{\varepsilon_t}{\beta_t(\theta)} \cdot \left(\cos\left(\beta_t(\theta)\right) - 1 + \varepsilon_t\right)\right]\right]$$

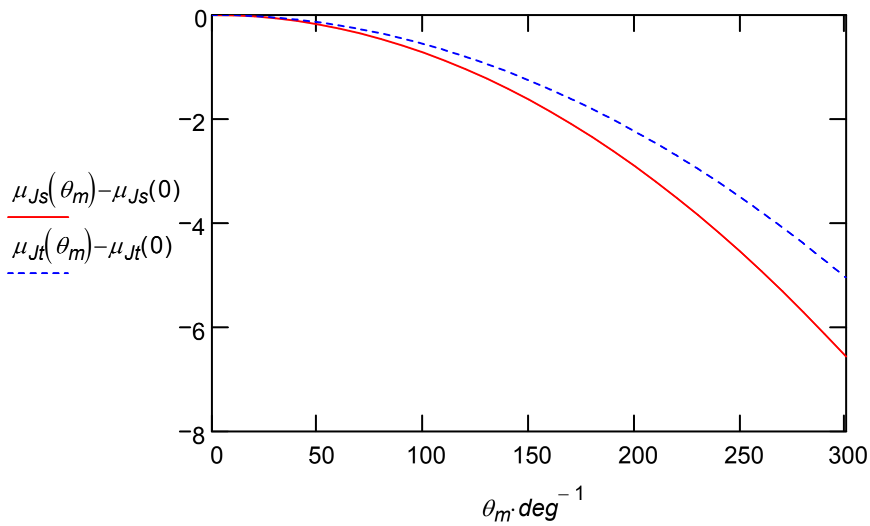
$$J_{st'}(\theta) := J_{st'}(\theta) + m_s \cdot L^2 \cdot \left(A(\theta) \cdot \frac{3 \cdot \varepsilon_t - 3 \cdot \varepsilon_t^{\,2} + \varepsilon_t^{\,3}}{3}\right) \qquad J_{st'}\left(\theta_0\right) = 0.53\, mg \cdot cm^2$$

$$J_t(\theta) := J_{sc}(\theta) + J_{st}(\theta) + J_{st'}(\theta) \qquad J_t\left(\theta_0\right) = 5.411\, mg \cdot cm^2$$



Perturbation de période

$$\delta_{Jt}\left(\theta_0\right) := \frac{1}{2 \cdot \pi \cdot J_b} \cdot \int_0^{\pi} J_t\left(\theta_0 \cdot \cos(\varphi)\right)\, d\varphi \qquad \mu_{Jt}\left(\theta_0\right) := -86400 \cdot \delta_{Jt}\left(\theta_0\right)$$



$$\mu_{Js}(0) = -642.61$$

$$\mu_{Jt}(0) = -610.627$$

$$\mu_{Js}\left(\theta_0\right) = -647.918$$

$$\mu_{Jt}\left(\theta_0\right) = -614.71$$

$t_f = 1.6$